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Exponential Filled Function Method for Solving Multi-dimension Global Optimization

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Abstract
The filled function technique is a highly efficient and successful approach for addressing global optimization problems. The performance of the filled function relies on parameters and attributes such as continuity, differentiability, and speed rate of reaching the optimal solution. This paper introduces a continuously differentiable filled function with a single parameter, and theoretical arguments are offered to demonstrate its features. A filled function method is built based on the suggested function to solve unconstrained global optimization issues. The numerical results of the suggested filled function on various test functions demonstrate the efficiency of this method.

1. Introduction

The general formula of the unconstrained global optimization problem can be given as follows:

$$(P) : \min_{x\in\Omega} f(x) \qquad (1)$$

where $f(x)$ is continuously differential objective function and $\Omega = \prod_{k=1}^{n}[l_i, u_i] \subset \mathbb{R}^n$ is a search domain area [1].

We present in this paper a single prameter filled function to find the solutions of the problem (P) as follows:

$$F(x, x_k^*, \rho) = \exp(-\rho\|x - x_k^*\|^2)r\big(f(x) - f(x_k^*)\big) \qquad (2)$$

where

$$r(t) = \begin{cases} 1 & t \geq 0, \\ 2 - \exp(-t) & t < 0, \end{cases}$$

$\rho > 0$ is parameter, $t = \big(f(x) - f(x_k^*)\big)$ and x_k^* is the current local minimizer so far.

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On the other side, $F(x, x_k^*, \rho)$ does not require to go more down in the interval when $f(x) < f(x_k^*)$, it just needs to locate any minimizer or stationary point that is used as an initial point to minimize $f(x)$ to locate more efficient solutions. The parameter ρ helps to find that point (stationary point) with the least time and least function evaluation which is important in global optimization problems formulation modelling.

Figure 1 explains how the filled function works and starts from the current local minimizer x_k^* , avoiding the higher minimizer x_{k+1}^* and finding a better solution x^* with the effects of the parameter ρ.

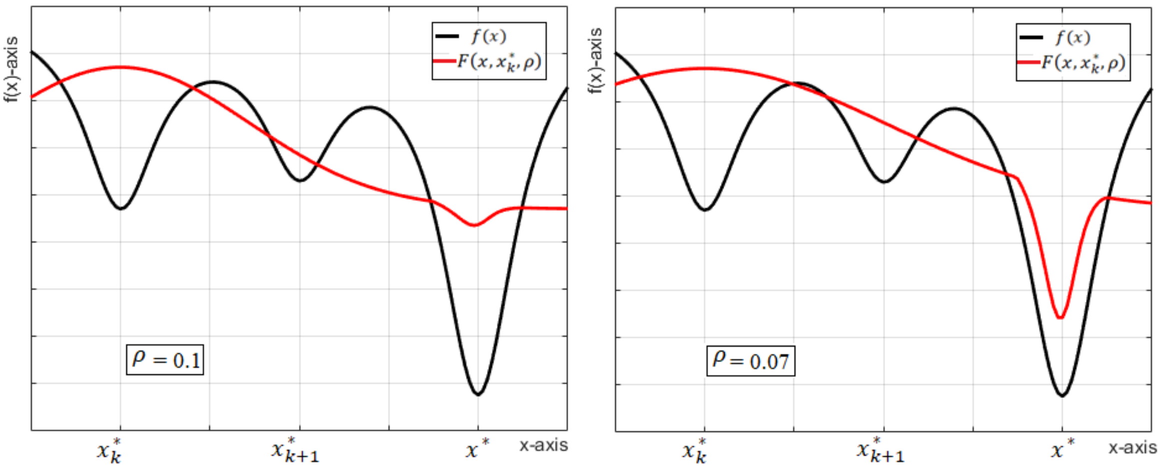

Figure 1. Illustration of the proposed filled function for different ρ.

A wide variety of real-life problems have been successfully and reliably resolved using global optimization techniques. They have been used in numerous scientific fields, including computer science and information technology, agriculture, geography, engineering, economics or even in everyday life such as planning a tourist trip [2-4].

According to theoretical studies and numerical experiments, the filled function method considers one of the most effective and successful methods for finding the solution to global optimization problems. The filled function approach is a deterministic method that uses deterministic ways to avoid the current local minimizer and instead discover other suitable solve. In 1987 Ge and Qin presented the first version of the filled function [5,6], and this function was developed by Xu and Wu [7-9]. The work of the filled function approach was discovered to provide answers to three logical queries: the first is how to identify the local minimizer, the second is how to leave the present local minimizer and locate other suitable solution, and the third is how to converge from the global minimizer point. Numerous investigations have been conducted to create the filled function approach as a result of these reasons., see [11-12].

2. Preliminaries

The following presumptions must be defined and used for the duration of this paper.

A1: Considering that Ω is a closed bounded set that includes all local minimizers of f, we describe $S_1 = \{x \in \Omega | f(x) \geq f(x_k^*), x \neq x_k^*\}$ and $S_2 = \{x \in \Omega | f(x) < f(x_k^*)\}$.

A2: The number of all values of the local minimum of the function f must be countable and limited.

A3: Suppose that $f(x)$ is coercive. Then, this aspect indicates that $f(x) \to +\infty \; as \; \|x\| \to +\infty$.

According to the above assumptions, we introduce the following definitions:

Definition 2.1 [11]: Assume that $x_k^* \subset \Omega$ is a local minimizer of f so far. The set $B_k^* \subset \Omega$ is named a basin of the function $f(x)$ at the point x_k^* if one of the local search methods going from each point in B_k^* gets x_k^*. The point x_{k+1}^* of f is greater or less than the minimizer x_k^* if $f(x_{k+1}^*) \geq f(x_k^*) \; or \; f(x_{k+1}^*) \leq f(x_k^*)$, the basin B_{k+1}^* at x_{k+1}^* which can be called greater or less than B_k^*.

Definition 2.2 [11]: Let x_k^* is a local minimizer of f. A function $F(x, x_k^*, \rho)$ is called a filled function of the objective function $f(x)$ at x_k^* if it meets the following criteria:

- x_k^* is a local maximizer of $F(x, x_k^*, \rho)$, and the whol basin B_k^* is a part of a hill of $F(x, x_k^*, \rho)$.
- $F(x, x_k^*, \rho)$ has no stationary point in all basins of $f(x)$ greater than B_k^*.
- If the point x_k^* is not a global minimizer of $f(x)$ in that case $f(x)$ has a basin lower than B_k^* and $F(x, x_k^*, \rho)$ must have a stationary point in that basin.

3. Theoretical Part

In this part, we demonstrate that $F(x, x_k^*, \rho)$ is a filled function and meets the conditions defined in Definition 2.2.

Theorem 1: Assume that x_k^* is a current local minimum found so far of the objective function $f(x)$ and $F(x, x_k^*, \rho)$ is a filled function at the point x_k^*, therefore x_k^* is a strict maximizer of the function $F(x, x_k^*, \rho)$ for all $\rho > 0$.

Proof: Since x_k^* is a local minimizer of the function $f(x)$ and B_k^* is a basin of x_k^*, therefore $\forall x \in B_k^*, x \neq x_k^*$, we have $f(x) > f(x_k^*)$ and $t > 0$.

So $F(x, x_k^*, \rho) = exp(-\rho\|x - x_k^*\|^2) < 1 = F(x_k^*, x_k^*, \rho)$, that is the first condition of Definition 2.2 is achieved and x_k^* is a local maximizer of the function $F(x, x_k^*, \rho)$.

Theorem 2: Let x_k^* is a local minimum point of $f(x)$, and for any point x belongs to the set S_1, the function $\nabla F(x, x_k^*, \rho) \neq 0$ for each $\rho > 0$.

Proof: By using any point $x \in S_1 = \{x \in \Omega | f(x) \geq f(x_k^*), x \neq x_k^*\}$, we have $t \geq 0$ that means $F(x, x_k^*, \rho) = exp(-\rho\|x - x_k^*\|^2)$ and

$$\nabla F(x, x_k^*, \rho) = -2\,\rho(x - x_k^*)\exp(-\rho\|x - x_k^*\|^2) \neq 0.$$

That mean for the whole the set S_1, the function $F(x, x_k^*, \rho)$ lacks a stationary point.

Theorem 3: If the current local minimizer x_k^* is not a global of $f(x)$, then the function $F(x, x_k^*, \rho)$ has at least one local minimum point in the set $S_2 = \{x \in \Omega | \, f(x) < f(x_k^*)\}$.

Proof: Since the point x_k^* is not a global munimum, then $f(x)$ has another local minimum point in S_2. Since $F(x, x_k^*, \rho)$ is a smooth function (continuously differentiable) on $\Omega \subset \mathbb{R}^n$, then $F(x, x_k^*, \rho)$ has a local minimum point when $f(x) < f(x_k^*)$, say x'. On the other hand, because $F(x, x_k^*, \rho)$ is differentiable at x' and this point is a local minimum of $F(x, x_k^*, \rho)$, then $\nabla F(x', x_k^*, \rho) = 0$. We know that S_2 is non-empty, that means there exists at least one point $z' \in S_2$ such that $F(z', x_k^*, \rho) < 0$. Thus $F(x', x_k^*, \rho) \leq F(z', x_k^*, \rho) < 0$ so $x' \neq x_k^*$. We know $x' \notin S_1 = \{x \in \Omega | \, f(x) \geq f(x_k^*), x \neq x_k^*\}$, from Theorem 2; therfor $x' \in S_2$.

3.1. Filled function algorithm

Finally, in this part we propose a filled function algorithm in the manner described below based on the previous information:

Step 1: Set $k = 1, \rho > 0$, $\varepsilon = 10^{-2}$, choose directions d_i $(i = 1,2, \dots, n)$, and $x_{int} \in \Omega$ as an initial point.

Step 2: Set $i = 1$, use x_{int} as a start point to minimizing the objective function $f(x)$ and find any local minimum point x_k^*.

Step 3: Construct the proposed filled function $F(x, x_k^*, \rho)$ at the current local minimum point x_k^* so far:

$$F(x, x_k^*, \rho) = \exp(-\rho \|x - x_k^*\|^2) r\big(f(x) - f(x_k^*)\big)$$

Step 4: If $i \leq N$, take $x = x_k^* + \varepsilon d_i$ and run (Step 5); otherwise run (Step 6).

Step 5: Minimize $F(x, x_k^*, \rho)$ use the point x to find a local minimum point x_F of $F(x, x_k^*, \rho)$, if $x_F \in \Omega$ then setting $x_{int} = x_F$, $k = k + 1$ and run (Step 2); if not, set $i = i + 1$ and run (Step 4).

Step 6: Put $x^* = x_k^*$ as a global minimum point of $f(x)$ and stopping run the algorithm.

4. Numerical Experiments

In this part, our algorithm is tested on problems 1-10 which are listed in Table 1. On a computer with an Intel(R) Core(TM) (i7-3687U CPU and 2.60 GHz) and Matlab R2016a, we constructed our technique by evaluating 10 distinct beginning points separately for each of these problems, and these starting points are picked evenly from the domain Ω.

Table 1: The list of various test functions

Function (No.)	Dimension (n)	Functions	Optimum value	Interval
1	2	2-Dimensional function $c = 0.02$	$zero$	$[-a, a]^2, a = 3$
	2	2-Dimensional function $c = 0.2$	$zero$	$[-a, a]^2, a = 3$
	2	2-Dimensional function $c = 0.5$	$zero$	$[-a, a]^2, a = 3$
2	2	3-Hump back camel function	$zero$	$[-a, a]^2, a = 3$
3	2	6-Hump back camel function	-1.0316	$[-a, a]^2, a = 3$
4	2	The function of Treccani	$zero$	$[-a, a]^2, a = 3$
5	2	The function of Goldstein and Price	3.000	$[-a, a]^2, a = 3$
6	2	Shubert function	-186.73091	$[-a, a]^2, a = 10$
7	2	Rastrigin function	-2.0000	$[-a, a]^2, a = 3$
8	2	(RC)Branin function	0.3979	[-5, 10] × [10, 15]
9	2,3,5,7,10,15,20,30	Sin-square I function	$zero$	$[-a, a]^2, a = 10$
10	2,3,4,7,10,15,20,30	Levy function	$zero$	$[-a, a]^2, a = 10$

- The symbols mean which used in this part in tables are listed as follows:
- No: represents the test function problem numbers;
- n: represents the test function problem dimensions;
- k: the iteration number;
- x^*: the best solution of the objective function (global minimizer);
- F_E : the mean of functions evaluations of $f(x)$ *and* $F(x, x_k^*, \rho)$;
- Time: the mean of attempts time in 10 different initial points (second);
- F_M : the mean of the objective function values in the 10 attempts;
- F_B : the best value of the objective function in the 10 attempts;
- Ratio: the proportion of success to find a global minimizer using 10 different points as starting points;

The results of our algorithm on problems 1-10 are introduced in Table 2 for solving two dimension and on problems 9,10 are given in Table 3 for solving different dimensions. Our proposed method is evaluated on ten test problems with dimensions ranging from one to thirty, and each problem is tested on ten uniform trail points as a starting point.

Table 2: The computational results of our proposed method on problems 1-10

No.	n.	k	F_E	x^*	F_M	F_B	Time	Ratio %
1	2(c=0.05)	2	182	(0.9820; -0.0565)	2.0689e-13	6.6609e-16	0.0222	100
	2(c=0.2)	3	213	(1.8973; -0.3005)	1.8332e-11	2.7425e-15	0.0987	100
	2(c=0.5	2	227	(1.8513; -0.4021)	1.2796e-11	3.3378e-15	0.1307	100
2	2	1	405	(0; 0)	1.3818e-10	0	0.0513	100
3	2	2	182	(-0.0898; -0.7127)	-1.0316	-1.0316	0.0212	100
4	2	2	111	(0; 0)	3.7911e-10	1.1915e-16	0.0144	100
5	2	1	460	(0; -1)	3.0000	3.0000	0.0534	100
6	2	2	466	(-1.4251; 5.4829)	-186.7309	-186.7309	0.0617	100
7	2	1	725	(0; 0)	6.3949e-15	0	0.1066	100
8	2	1	213	(-3.1416; 12.2750)	0.3979	0.3979	0.0236	100
9	4	4	520	(1; 1)	1.1634e-13	.5887e-15	0. 0885	100
10	4	3	414	(1; 1)	4.6210e-13	2.6872e-17	0.0691	100

Table 3: Results of our algorithm for solving problems 9 and10 with different dimensions.

No.	n.	k	F_E	F_M	F_B	Time	Ratio%
9	3	4	879	3.6027e-12	1.2382e-16	0.0962	100
	5	5	2371	5.1466e-10	4.7481e-16	0.1937	95
	7	3	2138	1.9724e-13	2.8592e-15	0.2301	86
	10	3	5105	1.3818e-10	2.9901e-15	0.2573	82
	15	4	5377	2.9888e-06	6.8849e-14	0.2213	83
	20	3	9255	1.7091e-10	2.8532e-15	0.3139	87
	30	3	13500	5.4067e-11	7.3347e-16	0.4622	78
10	3	2	867	4.1314e-13	7.0160e-15	0.0811	100
	4	2	1120	3.4550e-13	2.1645e-15	0.1060	100
	7	3	1650	1.4773e-13	3.7005e-16	0.1230	93
	10	4	4112	6.4680e-13	1.2304e-14	0. 2825	92
	15	1	2588	4.4769e-14	1.2981e-14	0.1591	85
	20	2	7312	6.4947e-13	2.8522e-15	0.4276	95
	30	2	7990	6.4656e-13	3.0205e-15	0.4507	95

In Table 4 we compare our algorithm to the algorithms in [13] and [14], in terms of the value of iteration numbers k and the mean value of function evaluations F_E. It can be seen from Table 4 that the experiment results of our proposed method are more efficient than the methods introduced in [13] and [14].

Table 4: Comparison of our algorithm with other algorithms

		Our Algorithm		The Algorithm in [13]		The Algorithm in [14]	
No.	n.	k	F_E	k	F_E	k	F_E
1	2(c=0.05)	2	182	2	214	2	310
	2(c=0.2)	3	213	1	291	2	788
	2(c=0.5	2	227	2	414	3	977
2	2	1	405	1	411	2	577
3	2	2	182	2	234	2	279
4	2	2	111	1	217	2	265
5	2	1	460	3	488	-	-
6	2	2	466	4	814	3	635
7	2	1	725	1	501	2	315
8	2	1	213	1	222	-	-
9	4	4	520	3	743	3	549
	3	4	879	2	3027	2	1283
	5	5	2371	2	4999	2	5291
	7	3	2138	2	8171	2	12793
	10	3	5105	3	8895	2	33810
	20	3	9255	3	18242	2	96223
	30	3	13500	4	43232	4	376885

5. Conclusion

This paper proposes a new filled function for solving unconstrained global optimization problems. The proposed filled function is controlled by one adjustable parameter. It is clear from the numerical examples that the suggested strategy is more successful for global optimization problems. The proposed filled function method effectively solves multi-model global optimization problems, so we designed a corresponding algorithm. Moreover, in order to show the performance of the presented algorithm, We conducted several numerical experiments. The preliminary calculation results demonstrate the proposed method is effective and promising.

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